

## Ratio and Proportions

**Ratio:** Comparison of 2 or more Quantities.

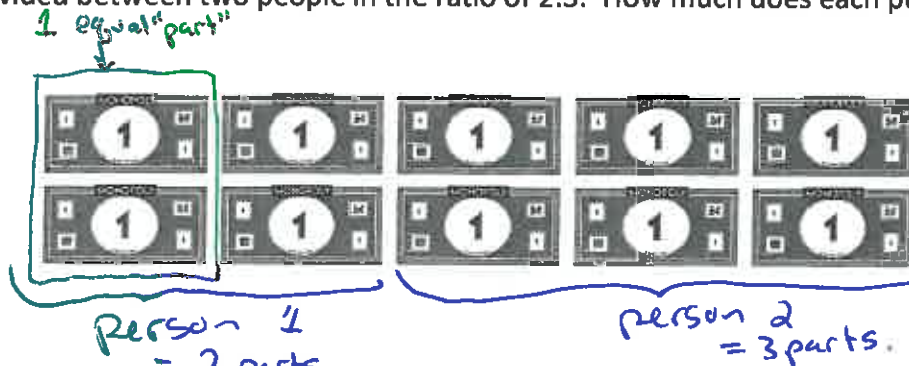
A ratio can be written in a variety of ways...

Example: The number of boys to girls in this classroom

8:12 or 8 to 12 or  $\frac{8}{12}$

### Using Ratios:

Example: \$10 is divided between two people in the ratio of 2:3. How much does each person get?



How many equal "parts" should the \$10 be divided into?  $2+3=5$  parts.

How big is each equal "part"?  $\frac{\$10}{5} = \$2$

How many equal "parts" does person 1 get? 2

Person 2? 3

How much \$ does person 1 get?  $2(\$2) = \$4$

Person 2?  $3(\$2) = \$6$

Example: \$160,000 is distributed to three people in the ratio of 2:3:5. How much does each person receive?

$$2+3+5 = 10 \text{ parts.}$$

$$\frac{\$160,000}{10} = \$16,000 \leftarrow 1 \text{ part}$$

$$\text{Person 1 } 2(16,000) = \$32,000$$

$$\text{Person 2 } 3(16,000) = \$48,000$$

$$\text{Person 3 } 5(16,000) = \$80,000$$

Example: The measures of the angles of a triangle are in the ratio of 2:3:4. Find the number of degrees in the smallest angle of the triangle.

$$2+3+4 = 9 \text{ parts}$$

$$\frac{180^\circ}{9} = 20^\circ \leftarrow 1 \text{ part}$$

$$m\angle 1 = 2(20^\circ) = 40^\circ$$

$$m\angle 2 = 3(20^\circ) = 60^\circ$$

$$m\angle 3 = 4(20^\circ) = 80^\circ$$

**Proportion:** two equal ratios.

Examples:

1.  $\frac{8}{2} = \frac{4}{1}$

2.  $5:25 = 10:50$

"means"  
"extremes"

3.  $\frac{2}{5} = \frac{12}{30}$

**Postulate:** In a proportion the product of the means = the product of the extremes.  
(means to cross-multiply)

Solve each of the following proportions:

To solve a proportion you must: Cross-multiply

1.  $\frac{5}{15} = \frac{a}{8+a}$

$$\begin{aligned} 15a &= 5(8+a) \\ 15a &= 40 + 5a \\ 10a &= 40 \\ a &= 4 \end{aligned}$$

2.  $x+11: x = 12: 16$

means  
extremes.

$$\begin{aligned} 12x &= 16(x+11) \\ 12x &= 16x + 176 \\ -4x &= 176 \\ x &= -44 \end{aligned}$$

3.  $\frac{x-4}{2} = \frac{3x}{x+4}$

$$\begin{aligned} 2(3x) &= (x-4)(x+4) \\ 6x &= x^2 - 16 \\ 0 &= x^2 - 6x - 16 \\ 0 &= (x-8)(x+2) \\ x-8=0 & \quad | \quad x+2=0 \\ x=8 & \quad | \quad x=-2 \end{aligned}$$

# Similar Polygons

Similar ( $\sim$ ) means: "Same Shape."

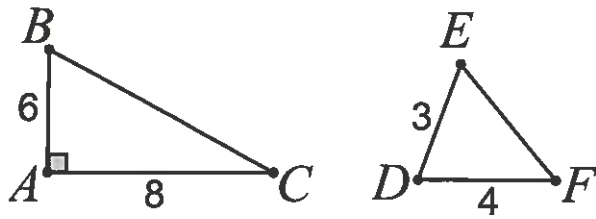
2 polygons are similar if:

- 1.) all corr.  $\angle$ 's are  $\cong$
- 2.) ratios of all corr. sides are proportional ( $=$ )

**Similarity Ratio:** Ratio of any two corr. sides of 2 similar polygons.  
Simplified

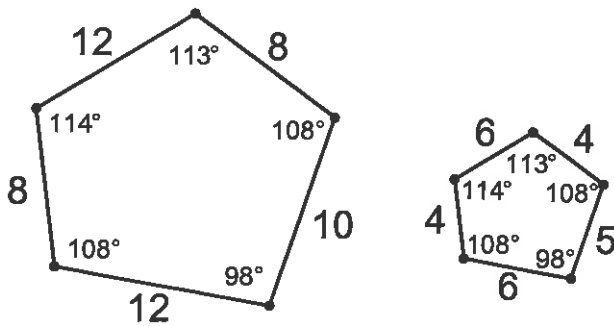
**Examples:** Are the polygons similar? If so, find the similarity ratio.

1.



no, since  $\angle A \not\cong \angle D$ .

2.



yes.

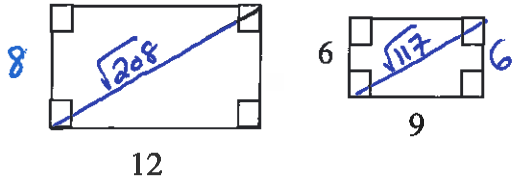
- all corr.  $\angle$ 's  $\cong$

- ratios all = .

$$\text{ie) } \frac{12}{6} = \frac{8}{4} = \frac{10}{5} = \frac{12}{6} = \frac{8}{4}$$

Similarity ratio:  $\frac{2}{1}$

**Example:** Given the similar rectangles:



Similarity Ratio =  $\frac{8}{6} = \left(\frac{4}{3}\right)$

Diagonals

Large =  $\sqrt{208} = 4\sqrt{13}$

Small =  $\sqrt{117} = 3\sqrt{13}$

Ratio =  $\frac{4\sqrt{13}}{3\sqrt{13}} = \left(\frac{4}{3}\right)$

Perimeters

Large = 40

Small = 30

Ratio =  $\frac{40}{30} = \left(\frac{4}{3}\right)$

Areas

Large = 96

Small = 54

Ratio =  $\frac{96}{54} = \frac{16}{9} = \left(\frac{4}{3}\right)^2$

**Theorem:** The ratio of any corresponding lengths of two similar polygons = Similarity Ratio.  
(Sides, Perimeter, Diagonals, Altitudes, Medians, etc.)

**Theorem:** The ratio of the areas of two similar polygons = (Similarity Ratio)<sup>2</sup>.

1. The ratio of the perimeters of two similar triangles is 9:12. ← Length.

a. What is the similarity ratio?  $\frac{9}{12} = \left(\frac{3}{4}\right)$

b. What is the ratio of their smallest sides?  $\frac{3}{4}$  Length

c. What is the ratio of their altitudes?  $\frac{3}{4}$  Length

d. What is the ratio of their medians?  $\frac{3}{4}$  Length

e. What is the ratio of their areas?  $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$  Area.

2. The ratio of the areas of two similar pentagons is 25:144. If the largest side of the smaller pentagon is 7, what is the length of the largest side of the larger pentagon?

$\frac{25}{144} = \frac{7^2}{x^2}$  ← Small sides.  
← Large sides.

$\frac{25}{144} = \frac{49}{x^2}$   
 $25x^2 = 7056$   
 $x^2 = \frac{7056}{25}$

$x = \sqrt{\frac{7056}{25}}$   
 $= \frac{84}{5} \approx 16.8$  units